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Effects of Variable Permeability On Unsteady Two-Dimensional Free-Convective Flow Through A Porous Medium Bounded By A Vertical Porous Surface

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Abstract : The effects of variable permeability on unsteady two-dimensional free-convective flow of a viscous incompressible fluid through a porous medium bounded by a vertical porous surface are investigated. The velocity and temperature distributions are derived, discussed and their profiles for various values of physical parameters are shown through graphs. Also the coefficient of skin-friction and coefficient of heat transfer at the surface are derived, discussed numerically and their numerical values for various values of physical parameters are presented through tables.

Key words : Variable permeability, Unsteady, free-convection, Porous medium.

Introduction

Free-convection problems has attracted considerable amount of interest because of its importance in atmospheric and oceanic circulation, nuclear reactors, power transfer etc. Porous medium is widely used to insulate heated body to maintain its temperature. It was considered to be useful in diminishing the natural free-convection, which would otherwise occur intensely on the vertical heated surface. To make the heat insulation of the surface more effective, it is necessary to study the free-convection flow through a porous medium and to estimate its effect on the heat transfer.

Study of the origin of the flow through porous medium release heavily upon Darcy's experimental law. By using Darcy's law Yamamoto and Yoshida (1974) considered a suction and injection flow with convective acceleration through a plane porous wall. They studied especially the vortex layer attached to the surface in the wall and the flow outside the vortex layer. Soundalgekar (1974) studied two-dimensional free-convective oscillatory flow past an infinite vertical porous plate with constant suction. Flow with convective acceleration through a porous medium was investigated by Yamamoto and Iwamura (1976). Cheng and Minkowycz (1977) studied the free-convection flow about a vertical flat plate embedded in a porous medium with application to heat transfer. Steady flow of a viscous fluid through a porous medium bounded by a porous surface with constant suction velocity by taking into account the presence of free-convection currents was investigated by Raptis *et.al.* (1981). Raptis and Singh (1985) have studied the free-convection flow past an impulsively started vertical plate in a porous medium by finite difference method. The effect of workable permeability on combined forced and free-convection in porous media was studied by Chandrashekhara and Namboodiri (1985). Jahagirdar and Lahurikar (1989) have studied the transient forced and free-convection flow past an infinite vertical plate. The free-convection effects on the flow of an

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ordinary viscous fluid past an infinite vertical porous plate with constant suction and constant heat flux were investigated by Sharma (1991). Baghel et.al. (1992) studied the effects of unsteady two-dimensional free-convective flow of a viscous incompressible fluid through a rotating porous medium. The steady two-dimensional flow through a porous medium bounded by a vertical infinite surface with constant suction velocity and constant heat flux in presence of free-convection current (where both velocity and temperature fields are constant along x-axis) was studied by Sharma (1992). The laminar free-convection flow of a micro-polar fluid past an arbitrary curved surface has been analyzed by Char and Chang (1995). Free-convection flow of a conducting micro-polar fluid with thermal relaxation including heat sources was analyzed by Ezzat (2004). Kumar (2007) investigated the unsteady two-dimensional free-convective flow of a viscous incompressible fluid through a rotating porous medium bounded by a vertical surface.

The aim of the present paper is to investigate the effects of variable permeability on two-dimensional free-convective unsteady flow of a viscous incompressible fluid through a porous medium bounded by a vertical porous surface of constant temperature. This surface absorbs the fluid with a constant velocity.

Formulation of the Problem

Consider unsteady two-dimensional flow of a viscous incompressible fluid through a porous medium of variable permeability, which is bounded by a vertical infinite porous surface. Let x^*-axis is taken along the surface in the upward direction and -axis is taken normal to it. The fluid properties are assumed constant except that the influence of density variation with temperature is considered only in the body force term. As the bounding surface is infinite in length, all the variables are functions of y^* and t^* only. Hence, by the usual boundary layer approximations, the governing equations of motion for unsteady flow of a viscous incompressible fluid through a porous medium are: y^*

Equation of continuity

$$\frac{\partial v^*}{\partial y^*} = 0 \Rightarrow v^* = -v_0 \qquad \text{..... (1.1)}$$

Equation of motion

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = \nu \frac{\partial^2 u^*}{\partial y^{*2}} + g\beta(T^* - T^*_\infty) - \frac{\nu}{K^*} u^* \qquad \text{..... (1.2)}$$

Equation of energy

$$\frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{\kappa}{\rho C_p} \frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\nu}{C_p}\left(\frac{\partial u^*}{\partial y^*}\right)^2 \qquad \text{..... (1.3)}$$

where u^* and v^* are the corresponding components of velocity along and perpendicular to the surface, v_0 the cross-flow velocity, ν the kinematic viscosity, g the acceleration due to gravity, β the coefficient of volume expansion for the heat transfer, the fluid temperature, the free stream temperature, κ the thermal conductivity, ñ the density of fluid, C_p the specific heat at constant pressure and the permeability of porous medium which is variable.

The boundary conditions are:

$$y^* = 0 : u^* = 0, T^* = T_w^* ;$$
$$y^* \to \infty : u^* \to \infty, T^* \to T_\infty^* , \qquad ..... (1.4)$$

where is the temperature of the surface.

Method of Solution

Introducing the following non-dimensional quantities

$$u = \frac{u^*}{v_0},\ y = \frac{v_0 y^*}{\nu},\ \Pr = \frac{\rho \nu C_p}{\kappa},\ T = \frac{(T^* - T_\infty^*)}{(T_w^* - T_\infty^*)},\ K = \frac{v_0^2 K^*}{\nu^2},$$

$$Gr = \frac{\nu g \beta (T_w^* - T_\infty^*)}{v_0^3},\ Ec = \frac{v_0^2}{C_p (T_w^* - T_\infty^*)},\ t = \frac{t^* v_0^2}{\nu} \qquad ..... (3.1)$$

into the equations (1.2) and (1.3) and using equation (1.1), we get the following set of differential equations:

$$\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} - \frac{\partial u}{\partial t} = -GrT + \frac{u}{K}, \qquad ..... (3.2)$$

K_w^*

$$\frac{\partial^2 T}{\partial y^2} + \Pr \frac{\partial T}{\partial y} - \Pr \frac{\partial T}{\partial t} = -\Pr Ec \left(\frac{\partial u}{\partial y} \right)^2, \qquad ..... (3.3)$$

and the corresponding boundary conditions are:

$$y = 0 : u = 0, T = 1;$$
$$y \to \infty : u \to 0, T \to 0, \qquad ..... (3.4)$$

where Pr is the Prandtl number, Gr the Grashoff number and Ec the Eckert number.

In order to solve the differential equations (3.2) and (3.3), we assume that

$$u = u_0(y) + \varepsilon e^{i\omega t} u_1(y)$$
$$T = T_0(y) + \varepsilon e^{i\omega t} T_1(y) \qquad ..... (3.5)$$

We have assumed that permeability of the porous medium is variable so

$$K = K_0 (1 + \varepsilon e^{i\omega t}), \qquad ..... (3.6)$$

where $K_0 > 0$ corresponds to permeability of the porous medium.

On substituting equations (3.5) and (3.6) into equations (3.2) and (3.3) and equating the coefficients of like powers of ε, we get the following set of differential equations:

$$\frac{d^2u_0}{dy^2}+\frac{du_0}{dy}-\frac{1}{K_0}u_0=-GrT_0 \qquad \text{..... (3.7)}$$

, ... (3.8)

$$\frac{d^2T_0}{dy^2}+Pr\frac{dT_0}{dy}=-Pr\,Ec\left(\frac{du_0}{dy}\right)^2, \qquad \text{..... (3.9)}$$

$$\frac{d^2T_1}{dy^2}+Pr\frac{dT_1}{dy}-i\omega\,Pr\,T_1=-2\,Pr\,Ec\left(\frac{du_0}{dy}\right)\left(\frac{du_1}{dy}\right), \qquad \text{..... (3.10)}$$

and the corresponding boundary conditions are:

$$y=0: u_0=0, u_1=0, T_0=1, T_1=0;$$

$$y\to\infty: u_0\to 0, u_1\to 0, T_0\to 0, T_1\to 0 \qquad \text{..... (3.11)}$$

In order to obtain the solutions of above coupled differential equations from (3.7) to (3.10), we expand u_0, u_1, T_0 and T_1 in powers of Eckert number Ec, assuming that it is very small

$$u_0(y)=u_{00}(y)+Ec\;u_{01}(y)+O(Ec^2),$$

$$\frac{d^2u_1}{dy^2}+\frac{du_1}{dy}-\left(i\omega+\frac{1}{K_0}\right)u_1=-\frac{d^2u_0}{dy^2}-\frac{du_0}{dy}-Gr(T_0+T_1)$$

$$u_1(y)=u_{10}(y)+Ec\;u_{11}(y)+O(Ec^2),$$

$$T_0(y)=T_{00}(y)+Ec\;T_{01}(y)+O(Ec^2),$$

$$T_1(y)=T_{10}(y)+Ec\;T_{11}(y)+O(Ec^2). \qquad \text{..... (3.12)}$$

Substituting (3.12) into equations (3.7) to (3.10), equating the coefficients of like powers of Ec and neglecting the higher order terms of Ec, we get

$$\frac{d^2u_{00}}{dy^2}+\frac{du_{00}}{dy}-\frac{1}{K_0}u_{00}=-GrT_{00}, \qquad \text{..... (3.13)}$$

$$\frac{d^2u_{01}}{dy^2}+\frac{du_{01}}{dy}-\frac{1}{K_0}u_{01}=-GrT_{01}, \qquad \text{..... (3.14)}$$

$$\frac{d^2u_{10}}{dy^2}+\frac{du_{10}}{dy}-\left(i\omega+\frac{1}{K_0}\right)u_{10}=-\frac{d^2u_{00}}{dy^2}-\frac{du_{00}}{dy}-Gr(T_{00}+T_{10}), \qquad \text{..... (3.15)}$$

$$\frac{d^2u_{11}}{dy^2}+\frac{du_{11}}{dy}-\left(i\omega+\frac{1}{K_0}\right)u_{11}=-\frac{d^2u_{01}}{dy^2}-\frac{du_{01}}{dy}-Gr(T_{01}+T_{11}), \qquad ..... (3.16)$$

$$\frac{d^2T_{00}}{dy^2}+\Pr\frac{dT_{00}}{dy}=0, \qquad ..... (3.17)$$

$$\frac{d^2T_{01}}{dy^2}+\Pr\frac{dT_{01}}{dy}=-\Pr\left(\frac{du_{00}}{dy}\right)^2, \qquad ..... (3.18)$$

$$\frac{d^2T_{10}}{dy^2}+\Pr\frac{dT_{10}}{dy}-i\omega\Pr T_{10}=0, \qquad ..... (3.19)$$

$$\frac{d^2T_{11}}{dy^2}+\Pr\frac{dT_{11}}{dy}-i\omega\Pr T_{11}=-2\Pr\left(\frac{du_{00}}{dy}\right)\left(\frac{du_{10}}{dy}\right), \qquad ..... (3.20)$$

with the corresponding boundary conditions:

$$y=0: u_{00}=0, u_{01}=0, u_{10}=0, u_{11}=0, T_{00}=1, T_{01}=0, T_{10}=0, T_{11}=0;$$

$$y\to\infty: u_{00}\to 0, u_{01}\to 0, u_{10}\to 0, u_{11}\to 0, T_{00}\to 0, T_{01}\to 0, T_{10}\to 0, T_{11}\to 0. \qquad ..... (3.21)$$

Now, the equations from (3.13) to (3.20) are ordinary coupled linear differential equations with constant coefficients. Therefore their solutions under the boundary conditions (3.21) are known and given by

$$+L_{13}\exp((R_4-\Pr)y),$$

$$u_{00}=L_1\exp(R_4y)+L_2\exp(-\Pr y) \qquad ..... (3.22)$$

..... (3.23)

$$u_{10}=-L_9\exp(R_6y)+L_7\exp(-\Pr y)+L_8\exp(R_4y), \qquad ..... (3.24)$$

$$\begin{aligned}u_{11}=&-L_{29}\exp(R_6y)+L_{21}\exp(-\Pr y)+L_{22}\exp(2R_4y)+L_{23}\exp(-2\Pr y)\\&+L_{24}\exp((R_4-\Pr)y)+L_{25}\exp(R_8y)+L_{26}\exp((R_4+R_6)y)\\&+L_{27}\exp((R_6-\Pr)y)+L_{28}\exp(R_4y),\end{aligned} \qquad ..... (3.25)$$

$$T_{00}=\exp(-\Pr y) \qquad ..... (3.26)$$

, ... (3.27)

$$T_{10}=0, \qquad ..... (3.28)$$

$$T_{11} = -L_{20}\exp(R_8 y) + L_{15}\exp((R_4 + R_6)y) + L_{16}\exp((R_4 - Pr)y) + L_{17}\exp(2R_4 y) + L_{18}\exp((R_6 - Pr)y) + L_{19}\exp(-2\,Pr\,y), \quad \text{..... (3.29)}$$

where R_4, R_6, R_8 and L_1 to L_{29} are constants, not presented here for the sake of brevity.

Coefficient of Skin-Friction

The coefficient of skin-friction at the vertical porous surface is given by

$$C_f = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \left(\frac{\partial u_0}{\partial y}\right)_{y=0} + \varepsilon e^{i\omega t}\left(\frac{\partial u_1}{\partial y}\right)_{y=0} \quad \text{..... (4.1)}$$

Coefficient of Heat Transfer

The rate of heat transfer in terms of Nusselt number at the vertical porous surface is given by

$$N_u = -\left(\frac{\partial T}{\partial y}\right)_{y=0} = -\left[\left(\frac{\partial T_0}{\partial y}\right)_{y=0} + \varepsilon e^{i\omega t}\left(\frac{\partial T_1}{\partial y}\right)_{y=0}\right] \quad \text{..... (5.1)}$$

Results and Discussion

It is observed from fig.1 that magnitude of the fluid velocity in case of steady flow is less than that of unsteady flow. In case of unsteady flow, the fluid velocity increases with the increase of the frequency of the fluid, Eckert number and the permeability parameter; while it decreases due to the increase of the phase angle and Prandtl number.

Fig.2 depicts that magnitude of the fluid velocity in case of steady flow is less than that of unsteady flow. In case of unsteady flow, the fluid velocity increases with the increase of Eckert number and the permeability parameter; while it decreases due to the increase of the frequency of the fluid, phase angle and Prandtl number.

It is also noticed that the magnitude of fluid velocity increases when the vertical surface is externally cooled as compared fig.2 with fig.1.

Fig.3 shows that magnitude of fluid velocity in case of steady flow is more than that of unsteady flow. In case of unsteady flow, the fluid velocity increases with the increase of the frequency of the fluid, phase angle and Prandtl number; while it decreases due to the increase of Eckert number and the permeability parameter.

Same behaviour is observed in fig.4 as in fig.3, but the magnitude of fluid velocity decreases when the vertical surface is externally heated.

It is observed from fig.5 that magnitude of fluid temperature in case of steady flow is less than that of unsteady flow. Further, in case of unsteady flow, it increases due to the increase of the permeability parameter and Eckert number; while it decreases due to the increase of the phase angle, frequency of the fluid and Prandtl number. Same behaviour is observed in fig.6 as in fig.5

Table1 shows that the coefficient of skin friction at the surface in steady flow is less than that of unsteady flow. Further, in case of unsteady flow, it increases with the increase of Eckert number and the permeability parameter; while it decreases due to the increase of Prandtl number, the frequency of the fluid and the phase angle.

Further the coefficient of heat transfer at the surface in steady flow is less than that of unsteady flow. In case of unsteady flow, it increases with the increase of Prandtl number, the frequency of the fluid and the phase angle; while it decreases due to the increase of Eckert number and the permeability parameter.

It is also observed that the coefficient of skin friction increases and coefficient of heat transfer decreases with the increase of Grashoff number.

Table2 shows reverse behaviour of the coefficient of skin friction as in Table1.

Table 1 : Values of the coefficient of skin-friction and the coefficient of heat transfer at the surface for various values of physical parameters, when the surface is externally cooled

ε	Pr	Ec	K	ω	ωt	C_f		Nu	
						Gr = 3	Gr = 5	Gr = 3	Gr = 5
0	0.71	0.025	3	0.2	π/6	3.133088	5.376641	-1.5077	-5.45023
0.25	0.71	0.025	3	0.2	π/6	3.303973	5.695131	-1.49954	-5.42757
0.25	7	0.025	3	0.2	π/6	0.415893	0.693409	6.982782	6.951723
0.25	0.71	0.05	3	0.2	π/6	3.367596	5.98968	-1.53406	-5.52346
0.25	0.71	0.025	5	0.2	π/6	3.635272	6.344574	-1.57322	-5.63224
0.25	0.71	0.025	3	0.4	π/6	3.294524	5.668169	-1.25248	-4.74149
0.25	0.71	0.025	3	0.2	π/3	3.255924	5.611578	-0.62719	-3.00434

Table 2 : Values of the coefficient of skin friction at the surface for various values of physical parameters, when the surface is externally heated

ε	Pr	Ec	K	ω	ωt
0	0.71	0.025	3	0.2	π/6
0.25	0.71	0.025	3	0.2	π/6
0.25	7	0.025	3	0.2	π/6
0.25	0.71	0.05	3	0.2	π/6
0.25	0.71	0.025	5	0.2	π/6
0.25	0.71	0.025	3	0.4	π/6
0.25	0.71	0.025	3	0.2	π/3

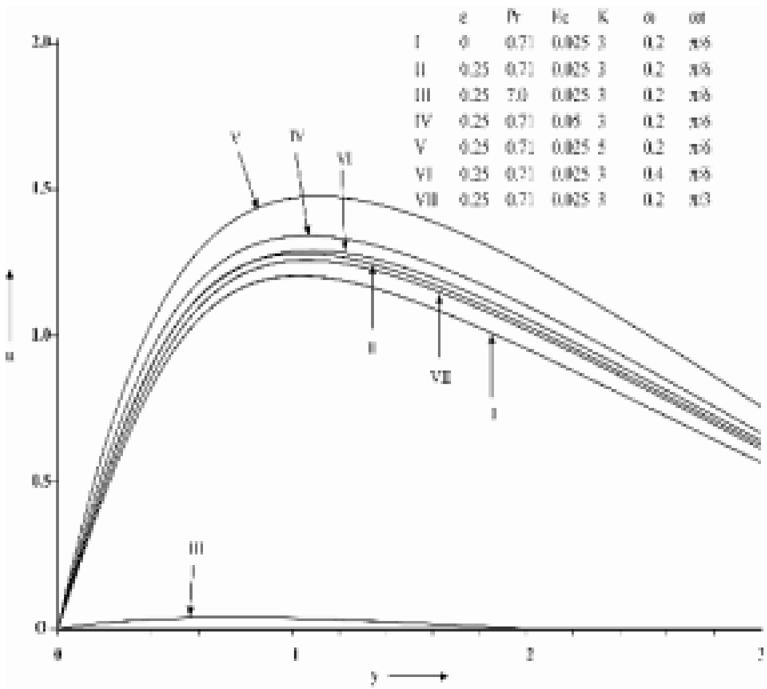

Fig. 1 : Velocity distribution versus y, when Gr=3

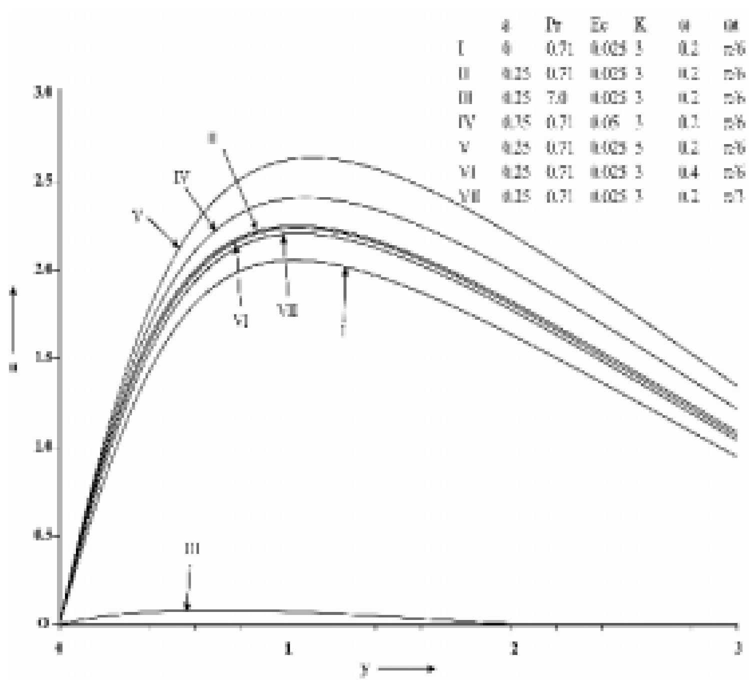

Fig. 2 : Velocity distribution versus y, when Gr=5

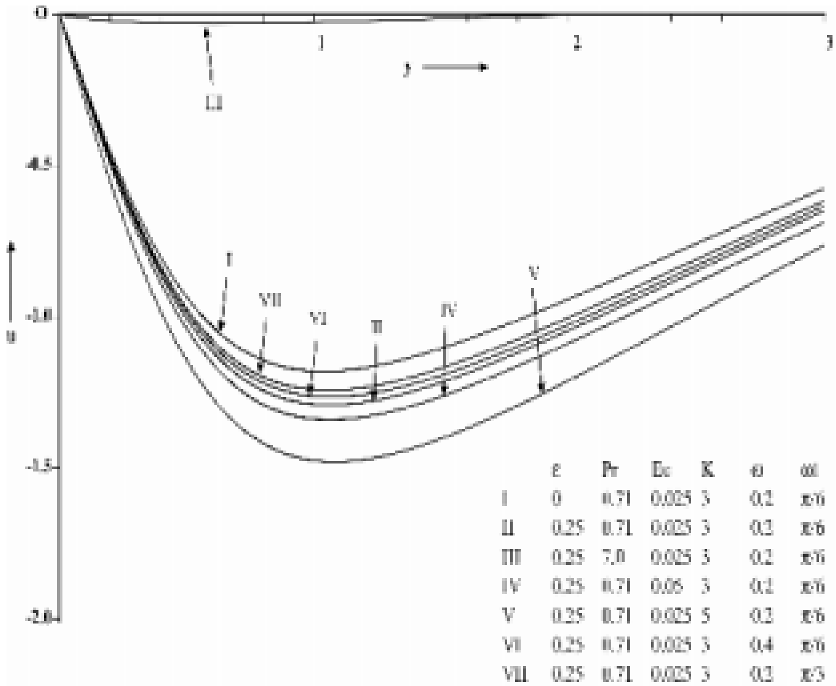

Fig. 3 : Velocity distribution versus y, when Gr= -3

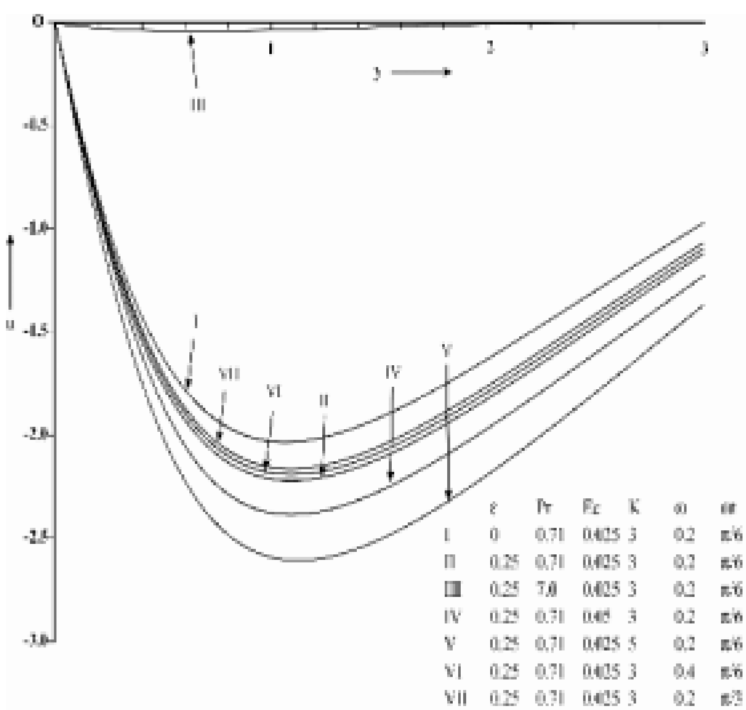

Fig. 4 : Velocity distribution versus y, when Gr= -5

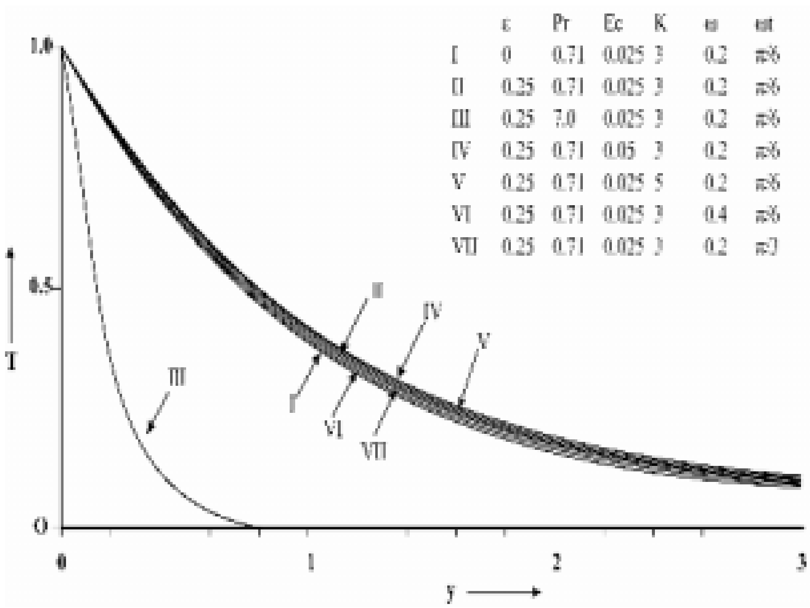

Fig. 5 : Temperature distribution versus y, when Gr= 3

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